

LETTERS TO THE EDITOR

PHYSICAL SCIENCES

New Source of Intense Magnetic Fields in Neutron Stars

RECENTLY there has been interest in the problem of intense magnetic fields (IMF) in gravitationally collapsed bodies¹⁻⁶. In particular, the radio emission from neutron stars (pulsars) suggests strong magnetic fields of the order of 10^{12} G (refs. 5 and 7). According to the law of flux conservation, in a collapse process the magnetic field strength increases as the square of the contracting factor a (>1). If the initial magnetic field is of the order of 10^3 G (the field in sunspots) before collapse, then assuming contraction from a star of one solar radius $\sim 10^{11}$ cm to that of a neutron star of 10^6 cm, a field strength of 10^{13} G may be achieved. Magnetic fields in ordinary stars are usually attributed to non-equilibrium processes such as the existence of currents in the form of drifting charges, but we have recently shown that a new kind of quasi-equilibrium state exists, which possesses a uniform self-consistent magnetization⁸.

Let us consider a degenerate electron gas of density ρ , in thermal equilibrium, and take the case when there are no currents in the form of drifting charges, that is, the local average velocity for electrons v_e and ions v_i is identically zero. Classically such an electron gas, of course, possesses no net magnetic field⁹.

We shall show, however, that in certain conditions a net magnetization can arise in an ideal degenerate electron gas at near zero temperature. The conditions arise from the previous magnetic history of the electron gas and the magnetization is due to the magnetic moment associated with Landau levels, that is, the energy states of charged particles in a magnetic field. We suggest the name LOFER (Landau Orbital Ferromagnetism) for this state of magnetization.

In brief, our external magnetic field H will induce a magnetic moment M in a degenerate electron gas. The value of M depends on the resultant field B

$$B = H + 4\pi M(B) \quad (1)$$

and because of the introduction of the Landau level by a magnetic field, $M(B)$ is not a simple but a highly spiked function of B (ref. 1). In the degenerate cases $4\pi M(B)$ is greater than H so that when the external field is removed the following equation is satisfied with a non-trivial value of B

$$B = 4\pi M(B) \quad (2)$$

Therefore, there can exist a state of macroscopic and quasi-stable magnetization in an ideal degenerate electron gas after the removal of the external field H , which is in the same direction as H .

For a degenerate electron gas the form of M has been obtained previously as¹

$$M/M_0 = \frac{1}{2} C_2(\mu) + \sum_{n=1}^{\infty} a_n^2 C_2(\mu/a_n) - \frac{B}{B_c} \sum_{n=1}^{\infty} n C_1(\mu/a_n)$$

$$C_1(x) = \ln(x + \sqrt{x^2 - 1}) \quad C_2(x) = \frac{1}{2} x \sqrt{x^2 - 1} - \frac{1}{2} C_1(x)$$

$$M_0 = \pi^{-2} \alpha B_c = 3.2637 \times 10^{10} \text{ G}, \quad B_c = \frac{m^2 c^3}{e \hbar} = 4.414 \times 10^{13} \text{ G}$$

$$a_n^2 = 1 + 2n\theta, \quad \theta \equiv B/B_c, \quad a_s \leq \mu < a_{s+1} \quad (3)$$

Here μ is the chemical potential plus the rest mass of the electron in units of mc^2 .

It has been shown that the quantity B in equation (3) is the magnetic induction, which is the sum of the contribution from a true current, H , as well as that due to the magnetization current contribution M , that is, $B = H + 4\pi M$. In our case, the true current contribution is 0 and $B = 4\pi M$. This results in a non-linear equation for M the solutions of which give the self-consistent magnetization M or, what is equivalent, the magnetic field B of the system. Only numerical solutions were possible. For a fixed density ρ , equation (3) admits many solutions for M , and there is a maximum value of B , B_{\max} which is a function of the electron density.

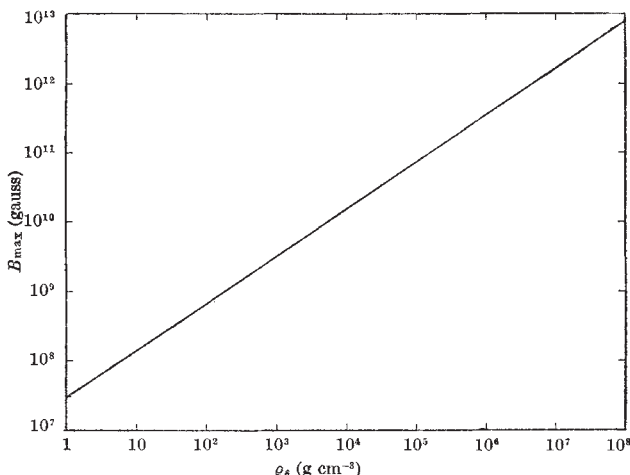


Fig. 1. The maximum self-consistent magnetic fields which are solutions of equation (3). The slope of the curve is $2/3$ as expected from the law of flux conservation.

The multiplicity of the solutions comes from the oscillatory behaviour of the function $M(B)$ which in turn is related to the discreteness of the energy eigenvalues. In Fig. 1 we only plot B_{\max} . Any value of $B < B_{\max}$ which satisfies equation (3) (for a given ρ) corresponds to a possible quasi-stable configuration⁸.

The remarkable feature of Fig. 1 is that $B_{\max} \propto \rho^{2/3}$ as expected from the law of flux conservation. The actual values of B_{\max} are from 10^8 G for $\rho_e \approx 1$ ($\rho_e = 10^{-6} \rho/\mu_e$) to 10^{13} G for $\rho_e \approx 10^9$. This agrees with the figures quoted before. The appearance of free neutrons at densities higher than 10^{10} g cm⁻³ will increase the matter density ρ (which in Fig. 1 has been computed without neutrons) up to a maximum of the order of 10^3 . At the currently accepted value of 10^{14} g cm⁻³ as the density of a neutron star, B_{\max} is of the order of 10^{13} G.

It has been said on many occasions that the magnetic field (due to flux conservation) of a freshly collapsed body is of the order of 10^{13} G. According to our theory a state of quasi-permanent magnetization may persist even after the current giving rise to the original field has been dissipated. Because this magnetization is likely to be along a particular direction, the magnetic field of a collapsed body is likely to be of the dipole type.

On our theory the LOFER field in a white dwarf is 10^7 G and in a neutron star 10^{13} G. The total number of extinct white dwarfs in our galaxy is almost as large as the total number of visible stars, and the total number of extinct neutron stars is about 100 times less. These bodies should therefore affect the propagation and generation of cosmic rays in the galaxy and the overall magnetic properties of our galaxy.

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Planetary Fission Events: Dynamical Constraints

PLANETARY fission has been proposed recently as a factor in the distribution of mass and density among the "terrestrial" planets. In particular, it has been suggested because of their similar densities that the Moon and Mars were once part of the Earth (see McCrea¹ and Lyttleton², although Lyttleton gave equal support to a capture hypothesis for the Moon). It has also been suggested that Mercury and Venus may have formed by the break-up of a single planetary mass³. Because changes in mass distribution in the solar system are subject to certain constraints, the limitations this may have placed on the past history of planetary fission have been evaluated.

A mass ejected from the Earth into a heliocentric orbit, as part of the new orbit, must return to a heliocentric distance corresponding to the launch point. This fact is well documented by the orbital parameters observed for the heliocentric deep space probes launched between 1958 and 1967 (Table 1). All have as either their aphelion or perihelion a distance close to the heliocentric distance of the Earth on the date of launch.

This principle can be applied to the hypothesis that Mars may once have been part of a common planetary mass with the Earth, and was ejected as a result of rotational instability. At present Mars has a semi-major axis of 1.52 AU so that initially, following separation from the Earth, the planet necessarily would be in a highly eccentric orbit ($e=0.33$) with the perihelion at 1 AU. The question is, are there any forces in the solar system which could convert such an orbit to the nearly circular orbit possessed by Mars today? The answer seems to be that there are no perturbing forces of sufficient magnitude to accomplish this task. This conclusion is reached by examining the parameters of the present orbit of Mars, which has an eccentricity of 0.09 and a minimum distance from the Earth of about 60×10^6 km, and by extrapolating backwards in time the perturbations produced by all the other visible masses in the solar system. In this way, the maximum possible displacement of Mars from its present orbit at any time in the past may be calculated. This type of analysis was first carried out by Stockwell⁴. These cal-

Table 1. ORBITAL PARAMETERS OF HELIOCENTRIC SPACECRAFT

Spacecraft	Launch date	Heliocentric		
		Earth distance at launch (AU)	Spacecraft orbit (AU)	
			Aphelion	Perihelion
Luna 1	Jan. 1959	0.99	1.32	0.98
Pioneer 4	March 1959	1.00	1.44	0.99
Pioneer 5	March 1960	1.00	0.99	0.81
Venus 1	Feb. 1961	0.99	1.02	0.72
Ranger 3	Jan. 1962	0.99	1.16	0.99
Zond 1	April 1962	1.00	1.00	0.65
Mariner 3	Nov. 1964	0.99	1.31	0.98
Mariner 4	Nov. 1964	0.99	1.43	0.99
Zond 2	Nov. 1964	0.99	1.39	1.00
Zond 3	July 1965	1.01	1.27	1.02
Pioneer 6	Dec. 1965	0.98	0.98	0.81
Pioneer 7	Aug. 1966	1.01	1.13	1.01
Mariner 5	June 1967	1.02	1.02	0.72
Pioneer 8	Dec. 1967	0.98	1.09	0.99
Pioneer 9	Nov. 1968	0.99	0.99	0.75

Spacecraft orbits and launch dates for the period 1958-68 were obtained from RAE Tech. Rep. 67059 (1967) by H. Hiller, from NASA Document N68-17909-3/31 and from NASA Tracking and Data Division, Washington, DC. Approximate heliocentric Earth distances at launch were derived using a value of ± 0.0056 AU per month from aphelion at 1.0167 AU on June 21, and truncating to the second decimal place. For the planetary fly-by missions, only the pre-encounter orbital parameters are given. The post-encounter orbits for the Mars and Venus probes were considerably modified during passage close to the planet. For example, the post-encounter orbit for Mariner 5 has a perihelion of 0.67 AU and aphelion of 0.65 AU. The reason for the apparent deviation between the calculated and expected values for Venus 1 is not known.

culations show that the closest the Martian mass has ever been to the Earth is 36×10^6 km and that the eccentricity of the orbit of Mars has never exceeded 0.14. It is concluded that Mars therefore cannot have originated by fission from the Earth.

These conclusions are valid for conditions in the solar system as they exist today. The calculations could undoubtedly be changed by intervention of some mass external to the system. This, however, is very improbable. A possible internal influence on planetary orbits would be the presence of large masses of dust and gas in the interplanetary medium. It is generally considered, however, that planetary accumulation was essentially complete and the interplanetary medium depleted before differentiation of the core and mantle of the Earth took place. Any contemporary influence of this type on the motions of the planets and comets has been effectively disproved⁵.

Table 2. CONSTRAINTS FOR PLANETARY FISSION

Planet	Maximum excursions of planetary orbit (AU)	Closest possible approach to inferior orbit (10^6 km)
Mercury	0.30-0.47	
(Luna)	0.47-0.63	0*
Venus	0.67-0.77	30
Earth	0.93-1.07	24
Mars	1.31-1.73	36
Asteroids	Not calculated	Overlap
Jupiter	4.9-5.5	48
Saturn	8.8-10.3	49
Uranus	17.7-20.4	111
Neptune	29.6-30.6	138
Pluto	29.6-49.5	Overlap

The maximum possible excursions of the planetary orbits were calculated from the data of Stockwell⁴. The figures demonstrate that only in two situations (the asteroids and Neptune-Pluto) is it possible for neighbouring planetary masses to have arisen by fission.

* The evidence for an encounter between Luna and Mercury has been outlined previously⁶.

By applying the same arguments, it is possible to draw up similar constraints for the other planetary masses. These are given in Table 2 where the maximum orbital excursions of the planets are listed, with an indication of possible overlap of the orbits in the past. In only two situations is there any possibility that separate masses now orbiting the Sun may at one time have been part of the same body. One of these situations, of course, is in the asteroid belt. Most of the asteroids, many of which are in highly eccentric orbits, pass through a point 2.8 AU from the Sun. This is fairly suggestive evidence that they